

Graph Minors and Homomorphism: Correlation

Overview of the course

In the theory of graphs there are two elementary operations: 1. Identifying a pair of adjacent vertices, 2. Identifying a pair of nonadjacent vertices. The first leads to theory of minors of graphs. The second leads to the theory of colourings and homomorphisms of graphs. In correlation between these two concepts, lies some of the most difficult questions of graph theory. For example, the four color theorem (in slightly stronger form) is to say that if one cannot identify adjacent pairs of a graph G to create a complete graph on 5 vertices, then one can find a sequence of identifying pairs of nonadjacent vertices to shrink the whole graph into four or less vertices. Proposing that 5 and 4 can be replaced by n and $n-1$, respectively, Hadwiger introduced a conjecture which is widely open and commonly believed to be the most difficult conjecture of graph theory.

In the first part, which is the introductory part of the course, we will do an introduction to classic study of minors and homomorphisms. In regard with minors, a characterization of planar graphs using forbidden minors will be presented. We will also see a decomposition theorem for the class of K_5 -minor free graphs. In regard with homomorphism several notions of colouring such as the chromatic number, fractional-chromatic number, circular-chromatic number will be presented. We will then learn two techniques for proving results on colouring planar graphs.

The second part of the course is an introduction to a new line of research: *the homomorphism of signed graphs*, which extends the classic notion of homomorphisms of graphs. Signed graphs provide a framework in which a stronger and more intuitive relation between the study of minor and homomorphism holds. While signed graphs are studied since 1930's for various considerations, the study of homomorphism of signed graphs has begun only recently. The study is motivated by the problem of mapping planar graphs into projective cubes. This problem, which we will discuss in depth, captures several well-known notions on colouring of planar graphs such as the four-colour theorem, the Grötzsch theorem, and the Jaeger-Zhang conjecture, it also relates to study of several colouring parameters of planar graphs such as the chromatic number, fractional and circular chromatic numbers and also to the edge-chromatic number.

Opportunity for Higher studies

As we look for input from young researchers, a good number of open problems of various difficulty will be presented. Furthermore, top student(s) of the class may have the chance to enrol in a Ph.D. or master program at University Paris 7 to do research on this subject. Support for such studies could be provided by Fondation Sciences Mathématiques de Paris (FSMP): <https://www.sciencesmaths-paris.fr/en/>, HOISGRA: an ANR-France grant for further research on homomorphisms of signed graphs, and by Institut de Recherche en Informatique Fondamentale, University Paris 7: <https://www.irif.fr/>

Modules	A: An introduction to classical notions : Jan 20 - Jan 25 B: Homomorphisms of signed graphs : Jan 27 - Feb 2 Number of participants for the course will be limited to thirty five.
You Should Attend If...	<ul style="list-style-type: none"> ▪ you are a mathematics /computer science research scientist or Masters student interested in cutting edge techniques of structural and algorithmic analysis. ▪ you are a faculty teaching/ working in combinatorics, graph/complexity theory interested in learning the fascinating theory of graph minors and the homomorphism of signed graphs . ▪ you are an industry person working on the development and analysis of algorithms on computationally hard problems.
Fees	<p>The participation fees for taking the course is as follows: Participants from abroad : US \$250. Student participants: INR 2,000. Faculty participants: INR 5,000. Industry: INR 8,000. Research Organizations: INR 8,000.</p> <p>The above fee include all instructional materials, computer use for tutorials and assignments, laboratory equipment usage charges, 24 hr free internet facility.</p> <p>Modes of payment: <u>Online transfer:</u> Account Name: CCE IIT Madras Acc. No: 3640111110 Branch: SBI, IIT Madras Branch, Chennai IFSC Code: SBIN0001055 Swift Code: SBININBB453</p> <p style="text-align: center;">OR</p> <p>Demand draft in favour of “CCE IIT Madras” payable at Chennai. The demand draft is to be sent to the course coordinator at the address given below.</p>
	<p>The participants may be provided with hostel accommodation, depending on availability, on payment basis. Request for hostel accommodation may be submitted through the link: http://hosteldine.iitm.ac.in/iitmhostel/</p>

The Faculty Members



Dr. Reza Naserasr is a researcher of CNRS-France, and a faculty member of Institut de Recherche en Informatique Fondamentale, Paris, France. Web Page: <https://www.irif.fr/~reza/>

His research interests include graph homomorphism and structural graph theory. He is fascinated by any combinatorial problem that is not easy to solve.



Dr. N Narayanan is a faculty member of the department of Mathematics, at The Indian Institute of Technology Madras, Chennai. His research interests include Graph Theory, Algebraic Combinatorics and Graph Ideals and anything that can be related to combinatorics.

Web Page: <https://home.iitm.ac.in/naru/>

GIAN Course Co-ordinator

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If you do not yet have a gian account, you are required to register paying a nominal fee at the page :

<http://www.gian.iitkgp.ac.in/GREGN>