

# Singular optimal control and dissipation inequalities

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## Overview

A basic problem in linear optimal control is to minimize the functional

$$J(u, t_0, t_1) = \int_{t_0}^{t_1} \begin{bmatrix} x \\ u \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt, t_0 < t_1 \leq \infty$$

subject to

$$E\dot{x} = Ax + Bu$$

and further constraints. In the classical case, when the matrices  $E$  and  $R$  are both nonsingular the minimization problem can be tackled via Hamiltonian matrices and Riccati equations. In the singular case, this is no longer possible. However, in general, the solvability of the minimization problem is equivalent to the existence of a function  $V$  which satisfies an inequality of the form

$$V(x(t_0)) - V(x(t_1)) \leq J(u, t_0, t_1). \quad (*)$$

A system with this property is called dissipative with respect to the function  $V$ . If  $V$  is of the form  $V(x) = x^T K x$  then  $(*)$  reduces to a linear matrix inequality (LMI). The Kalman-Yakubovich-Popov-Lemma gives necessary and sufficient conditions for the existence of such a matrix  $K$ . This Lemma is stated in several versions in the literature. One aim of the lectures series is to present a formulation that is as general as possible. In doing so, we review recent research results for optimal control of descriptor systems (i.e. the case when  $E$  is singular). Furthermore, we consider applications of this theory to robotics, electrical engineering, space navigation and flow control. Finally, we discuss pseudospectral methods, related current research topics and open research problems.

This course is organized in two modules that are encouraged to be taken together. The topics in *Module A* will expose the participants the solution set of algebraic Riccati equations, and a brief introduction to pseudospectra and computation of pseudospectra of a given matrix. In *Module B*, the nonsingular and singular optimal theory will be introduced. The topics in this module also include details of descriptor systems, dissipation inequalities and passivity, linear matrix inequalities, and a few open research problems. Applications of this theory to robotics, space navigation and flow control will also be discussed.

Course participants will learn these topics through lectures and numerical simulations. Assignments and problem solving sessions will be organized to stimulate research motivation of the participants.

<b>Modules</b>	<b>A: Algebraic Riccati Equations and Pseudospectra of Matrices</b> : December 6 - December 9
	<b>B: Singular Optimal Control</b> : December 12 - December 20, 2016

<p><b>You Should Attend If...</b></p>	<p><b>Number of participants for the course will be limited to fifty.</b></p> <ul style="list-style-type: none"> <li>▪ you are an electrical or a mechanical engineer or a research scientist interested in solving singular optimal theoretic problems that often arise in descriptor systems.</li> <li>▪ you are a mathematician or a student of mathematics interested to learn applications of linear algebraic methods in control theory and to understand the solution set of algebraic Riccati equations in purely linear algebraic way.</li> <li>▪ you are a student or faculty member from academic institution interested in learning the singular optimal control theory, linear matrix inequalities and its applications to engineering problems.</li> </ul>
<p><b>Fees</b></p>	<p>The participation fees for taking the course is as follows:</p> <p><b>Participants from abroad : US \$500</b>  <b>Industry/ Research Organizations: Rs. 20,000</b>  <b>Academic Institutions: Rs. 10,000</b></p> <p>The above fee includes all instructional materials, computer use for tutorials and assignments, 24 hr free internet facility. The participants will be provided with accommodation on payment basis.</p>

## The Faculty



**Dr. Michael Karowis** is a member of the research group 'Modelling, Numerical Analysis and Differential Equations' at the Technical University of Berlin, Germany. His research interests include Linear algebra and control theory, in particular, pseudospectra and stability radii,  $\mu$ -analysis, perturbation theory of eigenvalues, numerical range, and linear algebra over the quaternions.



**Dr. Bibhas Adhikari** is an Assistant Professor at the Indian Institute of Technology Kharagpur, India. His research interests include applied linear algebra, theory of complex networks and graph theoretic techniques in quantum information.

## Course Co-ordinator

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